DI-003-001617 Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

March-2022

Mathematic : Paper-BSMT-602(A)

(Mathematical Analysis-2 & Group Theory-2)

(Old Course)

Faculty Code : 003

Subject Code : 001617

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Write answer of each question in your main answer sheet.

1 Answer the following questions in briefly : **20**

(1) Define Principal ideal ring.

(2) Find characteristic of the ring (z, +,^{0})

(3) Define monic polynomial.

(4) Define Field.

(5) Find zero divisor of the ring $(z_6, +_6, 0_6)$

(6) Define Kernel of a homomorphism.

(7) If polynomial $g = (0, 5, -1, 2, 0, 0.......)$ than find degree of g.

(8) Define Division Ring.

(9) Define Ring with unity.

(10) State the first fundamental theorem of homomorphism.

(11) Define : Compact set.

(12) What is the greatest lower bound of set $\left\{\frac{1}{n} / n \in N\right\}$

(13) Find $L(e^t)$

(14) Determine whether the subset {0, 3} of metric space R is compact or not.

(15) Find $L(e^{2t}t)$

(16) Find $L^{-1}\left(\frac{1}{s-2}\right)$

(17) Show that R is not compact set.

(18) Define connected set.

(19) Determine whether set $\{1, 2, 3, ..., 11\}$ is connected or not.

(20) Find $L^{-1}\left(\frac{1}{S^2+1}\right)$

2 (A) Attempt any **three** : **6**

(1) Show that the sets A=(3, 4) and B(4, 5) are separated sets of metric space R.

(2) Show that subset R–{7} is not connected.

(3) Show that every finite subset of a metric space is compact.

(4) Find Laplace transform of $e^{-2t} \sin 5t$

(5) Prove that $L[3^{4t}] = \frac{1}{s - 4\log 3}$

(6) Find $L^{-1}\left(\frac{3s+4}{s^2+16}\right)$

2 (B) Attempt any **three** : **9**

(1) State and prove Bolzano-Weirstrass theorem.

(2) Prove that every singleton subset of any metric space is connected.

(3) If F is a closed subset of metric pace X and K is a compact subset of X Then prove that $F \bigcap K$ is also compact.

(4) If $L\{f(t)\} = F(s)$ then prove that $L[e^{at} f(t)] = F(s-a)$

(5) Find Inverse Laplace transform of $\log\left(\frac{s+b}{s+a}\right)$

(6) Find Laplace transform of $t^2 \sin 4t$

(C) Attempt any **Two** : **6**

(1) If (X, d) is a metric space E_1 and E_2 are connected subset of X and $E_1 \cap E_2 \neq \phi$ then prove that $E_1 \cup E_2$ is also connected.

(2) Prove that continuous image of connected set is connected.

(3) State and prove theorem of nested intervals.

(4) Prove that $L^{-1}\left(\frac{s}{\left(s^2+4\right)^2}\right) = \frac{1}{4} t \sin 2t$

(5) Using convolution theorem, find $L^{-1}\left\{\frac{s}{\left(s^2+4\right)^2}\right\}$.

3 (A) Attempt any **Three** : **6**

(1) For element a and b of a ring R, prove that $a0 = 0a = 0$

(2) If $\phi : (G,*) \to (G', \Delta)$ is Homomorphism. If N is a normal subgroup of G then prove that $\phi(N)$ is a normal subgroup of $\phi(G)$.

(3) Show that a cyclic group of order eight is homomorphism to a cyclic group of order four.

(4) (R, +) and (G, ×) are groups. $G = \{z \in C / |z| = 1\}$ then show that mapping $\phi : R \to G$ is homomorphism.

(5) $f(x) = (2, -3, 0, 2, 0, 0, ...,)$ and

$g(x) = (2, 4, 0, 0, 3, 0, ...) \in R[x]$ then find $f(x)+g(x)$.

(6) Let I be an ideal of a ring R with unity, Then prove that $| = R \; if \; 1 \in|$

(B) Attempt any **Three** : **9**

(1) Prove that a Homomorphism $\o : (G,*) \to (G', \Delta)$ is one-one iff $K\o = \{e\}$.

(2) Let $\o : (G,*) \to (G', \Delta)$ be a Homomorphism then prove that $K\o$ is a normal subgroup of G.

(3) Find all homomorphism's of (Z, +) onto (Z, +).

(4) State and prove factor theorem of polynomials.

(5) Give an example of left ideal which is not right ideal.

(6) In $R[x], f(x) = 4x^4 - 3x^2 + 1$ is divided by $g[x] = x^3 - 2x + 1$ then find quotient $q(x)$ and remainder $r(x)$

(C) Attempt any **Two** : **10**

(1) Show that $x^3 + 3x^2 - 8$ is irreducible over $Q[X]$

(2) State and prove division algorithm for polynomials.

(3) Find gcd of polynomials $f(X) = X^3 + 3x^2 + 3x + 3$ and $g(X) = 4x^3 + 2x^2 + 2x + 2$ of $Z_5[X]$ and express it of the form $a(X)f(X) + b(X)g(X)$

(4) State and prove first fundamental theorem of homomorphism.

(5) Prove that a field has no proper ideal.